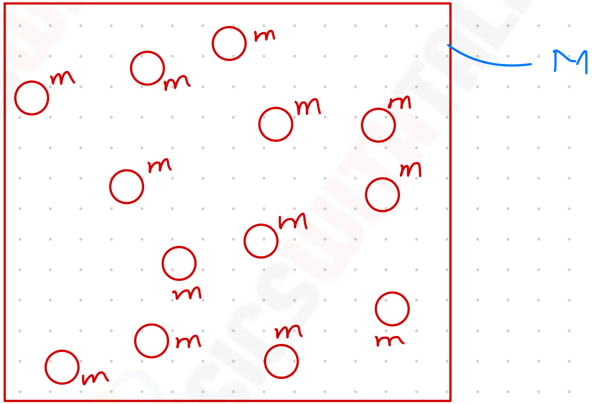
Ideal gases

PHYSICS WITH TALHA

The Mole

Suppose a gas is stored in a container



m → mass of each particle

M → mass of entire gas in the container

N → number of particles of gas

n → number of moles [unit : mol]

N_A → number of particles in one mole of a gas (Avagadro's constant)

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$$

We will look at gases made of a single element.
(e.g : He, Ar, Ne, Xe, Kr, Rn)

A_r → relative atomic mass : mass of one mole of a gas
(moler mass) [in grams]

________ × ________ × ________ × ____

$$\boxed{n = \frac{M}{A_r}}$$

e.g Argon gas has a moler mass of 40 g.
A container has 150 g of Argon gas. How many moles of the gas are there in the container?

$$n = \frac{150}{40} = 3.75 \text{ mol}$$

$$M = m \times N$$

$$A_r = m \times N_A \qquad N_A = 6.02 \times 10^{23}$$

if $n = \dfrac{M}{A_r}$

then $n = \dfrac{\cancel{m} \times N}{\cancel{m} \times N_A} \longrightarrow \boxed{n = \dfrac{N}{N_A}}$

Avagadro constant ($N_A = 6.02 \times 10^{23}$) : " the number of particles in 12 g of Carbon-12."

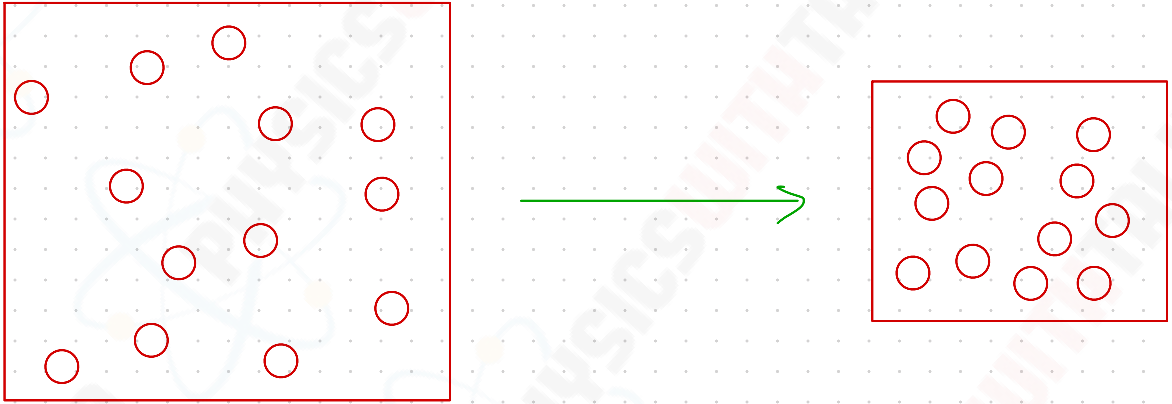
Mole : " amount of substance contains N_A (6.02×10^{23}) particles"

or " amount of substance containing the same number of particles as 12 g of carbon-12 ."

Gas laws

Three macro properties of gas are its pressure, volume, and temerature (in Kelvin)

1) Pressure vs Volume



P_1 , V_1

1000 Pa , 100 cm^3

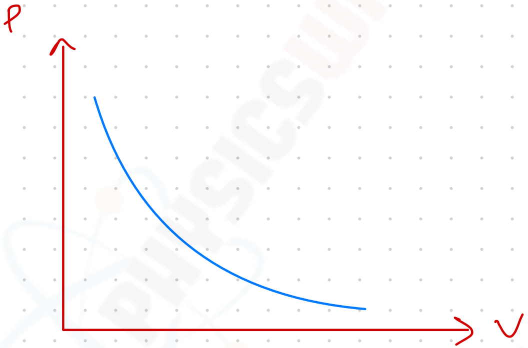
P_2 , V_2

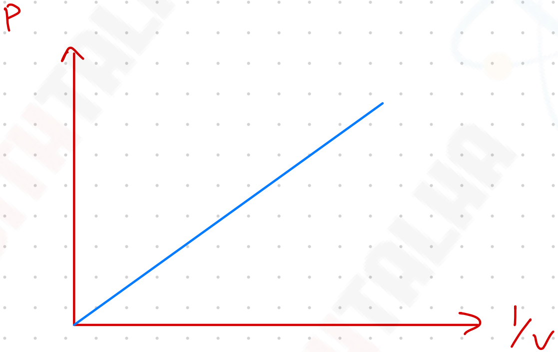
2000 Pa , 50 cm^3

Assumption : Temp constant , mass constant.

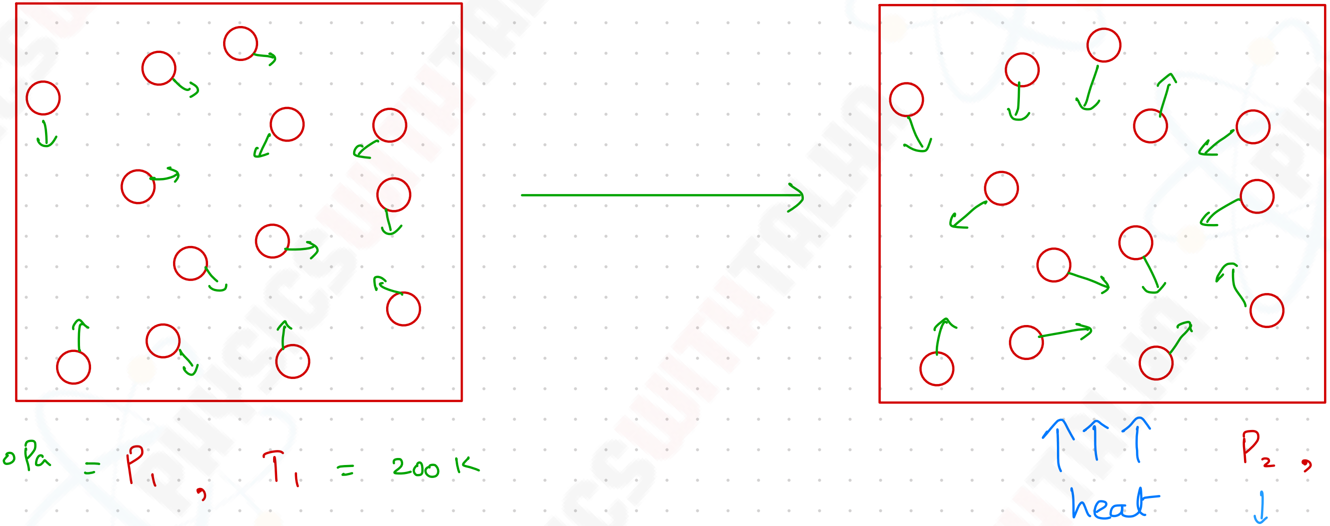
$$P \propto \frac{1}{V} \longrightarrow PV = \text{constant}$$

$$\boxed{P_1 V_1 = P_2 V_2}$$





2) Pressure vs Temperature (in K)



1000 Pa = P_1 , T_1 = 200 K

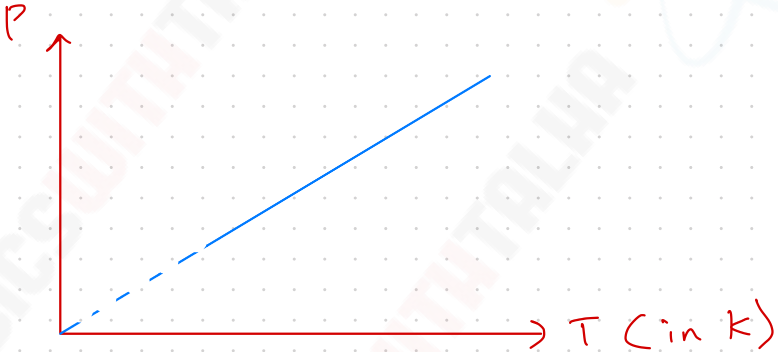
P_2 , T_2

2000 Pa , 400 K

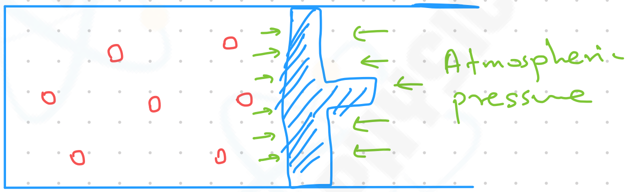
Assumptions: Volume constant, mass constant

$$P \propto T \longrightarrow \frac{P}{T} = \text{constant}$$

$$\boxed{\frac{P_1}{T_1} = \frac{P_2}{T_2}}$$

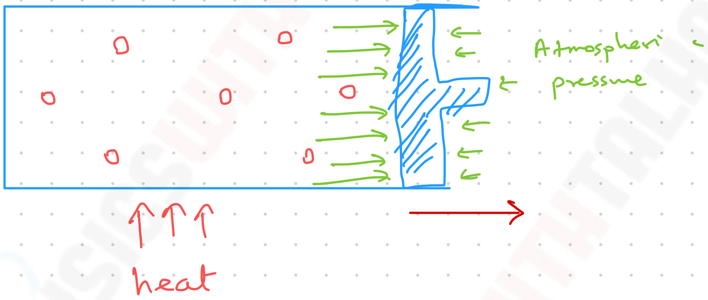


3) Temperature (in K) vs volume



V_1 , T_1

$100 cm^3$, 300K



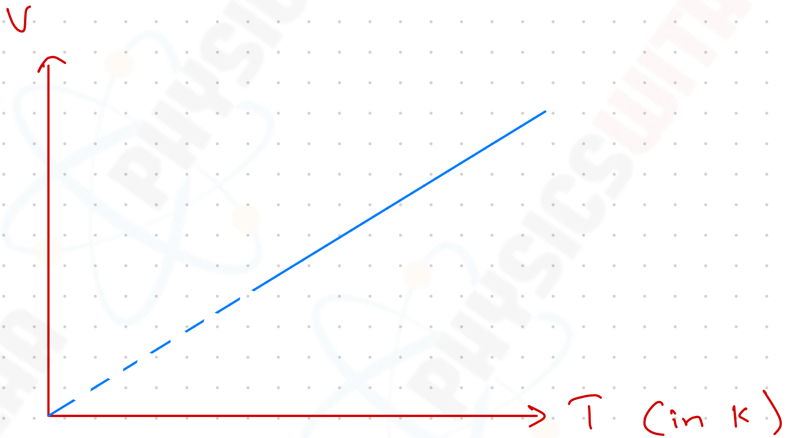
V_2 , T_2 600K

$200 cm^3$

Assumption : pressure constant , mass constant.

$$V \propto T \longrightarrow \frac{V}{T} = \text{constant}$$

$$\boxed{\frac{V_1}{T_1} = \frac{V_2}{T_2}}$$



→ if mass (or number of moles) of gas is constant.

$P_1V_1 = P_2V_2$, $\frac{P_1}{T_1} = \frac{P_2}{T_2}$, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$$\boxed{\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}}$$

Ideal gas equation

→ 3 gas law can be combined to form an equation

$$\boxed{PV = nRT}$$

$R = 8.31\ \text{J K}^{-1}\text{mol}^{-1}$ → universal gas constant

Temperature must be in Kelvin.

$$\boxed{T\ (\text{K}) = T\ (^\circ\text{C}) + 273.15}$$

→ we know that $n = \frac{N}{N_A}$

$$PV = \left(\frac{N}{N_A}\right) R\ T$$

R and N_A are both constants.

so $\frac{R}{N_A}$ is also a constant.

$$\boxed{\frac{R}{N_A} = K}$$ → Boltzmann constant

so $\boxed{PV = NkT}$ $k = 1.38 \times 10^{-23}\ JK^{-1}$

"Ideal gas is defined as a gas that obeys the ideal gas equation at all pressures, volumes, and temperatures."

→ Real gases are not ideal.

→ Ideal gas is just a model.

→ Real gases behave like ideal gas when:

1) Low density

2) Pressure is low

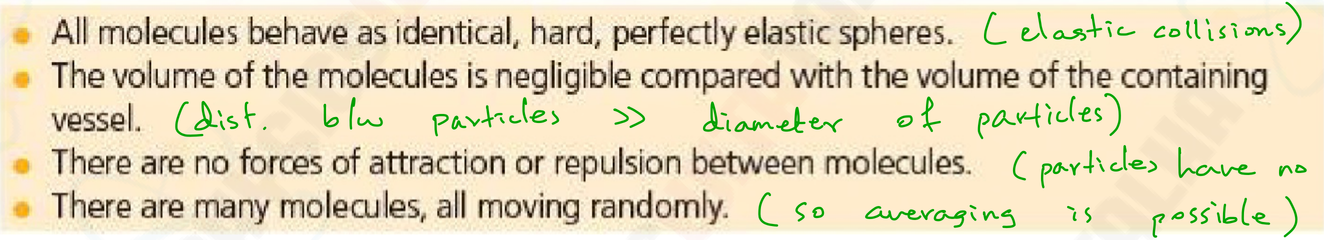
3) Temperatures are well above the liquification temperatures.

Kinetic Theory of gases

→ Gas is made up of tiny particles

→ The particles are always in motion.

→ There are 4 assumptions of kinetic theory of ideal gases.



1 - All molecules behave as identical, hard, perfectly elastic spheres. (elastic collisions)
2 - The volume of the molecules is negligible compared with the volume of the containing vessel. (dist. b/w particles >> diameter of particles)
3 - There are no forces of attraction or repulsion between molecules. (particles have no P.E)
4 - There are many molecules, all moving randomly. (so averaging is possible)

→ Consider a particle of mass, m, moving with horizontal component of velocity c_x.

→ When it hits the fall, the velocity would be equal and opposite after rebounding. (1)

$-c_x$

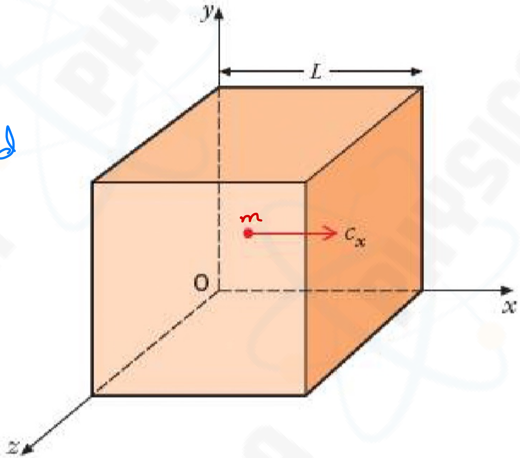


Figure 10.5 A gas molecule in a cubic container

→ To make a roud trip and collide with the same wall again, the time needed:

$$t = \frac{\text{dist}}{\text{speed}} = \frac{2L}{c_x}$$

→ The change in momentum (x-component) of the particle:

$$\Delta P = P_f - P_i = (-mc_x) - (mc_x) = -2mc_x$$

for magnitude only, neg. sign is ignored

The rate of change of momentum of the particle :

$$\frac{\Delta p}{\Delta t} = \frac{2mc_x}{t} = \frac{2mc_x}{\frac{2L}{c_x}} = \frac{m}{L} c_x^2$$

Force (x - comp.) exerted on (and by) the particle :

$$F = \frac{\Delta p}{\Delta t} = \frac{mc_x^2}{L}$$

→ If no. of particles is N, the total average force exerted in x- component is :

$$\text{Total } F = \frac{Nmc_x^2}{L}$$

$T_1 = 200°c$, 473 K

$T_2 = 210°c$, 483 K

$\Delta T = 10$ °c or K

→ Area of the wall

$$A = L^2$$

Pressure on the wall

$$P = \frac{\text{Total } F}{A} = \frac{\left(\frac{N m c_x^2}{L}\right)}{L^2} = \frac{N m c_x^2}{L^3}$$

Volume of container

$$V = L^3$$

So pressure becomes :

$$P = \frac{N m c_x^2}{V}$$

* Above expression is for x-component only. The particle will have x, y, and z component of velocity

$$c^2 = c_x^2 + c_y^2 + c_z^2$$

(Phythagoras Theorem)

Because we are dealing with a large number of particles and their motion is random (4), so we can assume their averages of squares are equal

$$\langle C_x^2 \rangle = \langle C_y^2 \rangle = \langle C_z^2 \rangle$$

So average square velocity will be :

$$\langle c^2 \rangle = \langle C_x^2 \rangle + \langle C_y^2 \rangle + \langle C_z^2 \rangle$$

$$= \langle C_x^2 \rangle + \langle C_x^2 \rangle + \langle C_x^2 \rangle$$

$$\langle C^2 \rangle = 3 \langle C_x^2 \rangle$$

mean square velocity

Rearranging :

$$\frac{\langle C^2 \rangle}{3} = \langle C_x^2 \rangle$$

So pressure becomes :

$$P = \frac{Nm\langle c^2\rangle}{3V} \longrightarrow \boxed{PV = \frac{Nm\langle c^2\rangle}{3}}$$

Total mass = Nm

Density , $\rho = \frac{\text{Total mass of gas}}{\text{Volume}} = \frac{Nm}{V}$

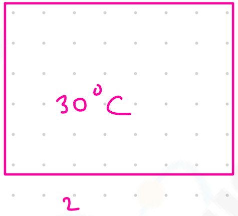
$$P = \frac{Nm\ \langle c^2\rangle}{3V}$$

So pressure becomes $\boxed{P = \rho\frac{\langle c^2\rangle}{3}}$

The avg K.E is :

$$\langle K.E \rangle = \frac{1}{2} m \langle c^2 \rangle$$

So total K.E is $= \frac{1}{2} N m \langle c^2 \rangle$



Ideal gas equation was : $PV = NkT$

So r.h.s of pressure equation and ideal gas equation can be equated

$$\cancel{N} kT = \frac{\cancel{N} m \langle c^2 \rangle}{3}$$

divide both sides by 2

$$\frac{1}{2} kT = \frac{1}{2} \times \frac{m \langle c^2 \rangle}{3}$$

$$\frac{1}{2} kT = \frac{\langle K.E \rangle}{3}$$

* Avg K.E of a molecule

$$\boxed{\langle K.E \rangle = \frac{3}{2} kT}$$

Total K.E $= \frac{3}{2} NkT$

Also ;

$$\frac{1}{\cancel{2}} m \langle c^2 \rangle = \frac{3}{\cancel{2}} KT$$

$$\sqrt{\langle c^2 \rangle} = \sqrt{\frac{3kT}{m}}$$

↳ root mean square velocity

$$\boxed{\text{r.m.s speed} = \sqrt{\frac{3kT}{m}}}$$

$$\text{r.m.s speed} \propto \sqrt{T}$$

$$\frac{\text{r.m.s speed}_A}{\sqrt{T_A}} = \frac{\text{r.m.s speed}_B}{\sqrt{T_B}}$$